Pitch Angle Restrictions in Late Type Spiral Galaxies Based on Chaotic and Ordered Orbital Behavior

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ABSTRACT

We built models for low bulge mass spiral galaxies (late type as defined by the Hubble classification) using a 3-D self-gravitating model for spiral arms, and analyzed the orbital dynamics as a function of pitch angle, going from 10° to 60°. Testing undirectly orbital self-consistency, we search for the main periodic orbits and studied the density response. For pitch angles up to approximately $\sim 20°$, the response supports closely the potential permitting readily the presence of long lasting spiral structures. The density response tends to "avoid" larger pitch angles in the potential, by keeping smaller pitch angles in the corresponding response. Spiral arms with pitch angles larger than $\sim 20°$, would not be long-lasting structures but rather transient. On the other hand, from an extensive orbital study in phase space, we also find that for late type galaxies with pitch angles larger than $\sim 50°$, chaos becomes pervasive destroying the ordered phase space surrounding the main stable periodic and quasi-periodic orbits and even destroying them. This result is in good agreement with observations of late type galaxies, where the maximum observed pitch angle is $\sim 50°$.

Subject headings: Chaos — galaxies: evolution — galaxies: kinematics and dynamics — galaxies: spiral — galaxies: structure

1. Introduction

It is assumed that the Hubble sequence reveals a strong correlation between the morphology of galaxies and their formation process. The study of this relation is a very active field where the merger hypothesis plays a fundamental role; however, it is very unlikely that

all trends observed in this sequence can be explained solely by this hypothesis. In particular, the long-term galactic dynamics of non-interacting galaxies is also determined by their inner structure. These highly non-linear stellar systems are prone to exhibit the coexistence of an astonishing dichotomy, an exquisite order right together with counterintuitive complex regions of chaos, where non-axisymmetric features such as spiral arms, bars, etc, play a key part.

Regarding to ordered motion, smooth and weak long-lasting large scale structures such as spiral arms (in general, low mass and/or low pitch angle), when modeled, present a relatively simple orbital structure made of families of quasi-periodic orbits surrounding the main periodic obits that sculpt spiral arms. The density response is also smooth and coincides nicely with the imposed potential in the case of low pitch angle spiral arms, for example. There are indications that even chaotic orbits could reinforce observed morphological features in this case (Kaufmann & Contopoulos 1996; Patsis, Athanassoula & Quillen 1997; Harsoula, Kalapotharakos & Contopoulos 2011). However, orbital analysis on dynamical models suggests that chaotic motion plays a significant role (Contopoulos 1983,1995; Contopoulos, Varvoglis & Barbanis 1987; Grosbøl 2003) in spirals.

Indeed, large scale structures are not expected to emerge on systems built out of pure chaos, this is, as long as chaos does not become pervasive, large scale structures of discs are practically unaffected. Recently, there has been an interesting discussion about the possible chaotic nature of the spiral structure (Patsis 2006; Voglis, Stavropoulos & Kalapotharakos 2006; Romero-Gómez et al. 2007; Voglis, Tsoutsis & Efthymiopoulos 2006; Contopoulos & Patsis 2008; Patsis et al. 2009).

Although the best known part of the Hubble classification regarding pitch angles, categorizes galaxies assuming that late types possess the most opened spiral arms (largest pitch angles), this is just the envelope of the classification. Late type spirals actually present a large scatter in this parameter (Kennicutt 1981; Ma et al. 2000), going from about 10° to 50°. In particular, late type spirals (Sb to Scd), are better fitted with strong spirals (Contopoulos & Grosbøl 1986; Patsis et al. 1991; Patsis, Grosbøl & Hiotelis 1997, and references therein), meaning they are far from being a slight perturbation that can be reproduced by the Lin & Shu (1967) spiral arm potential with a cosine function, solution of the Tight Winding Approximation (TWA).

We present a first result of a detailed orbital study on models of late type spiral galaxies as defined by Hubble (1926). In particular, this work is devoted to one of the structural parameters of spiral arms: the pitch angle. In this study some restrictions are imposed theoretically on their steady or transient nature, and on their maximum pitch angle.

This letter is organized as follows. In Section 2, the 3-D galactic potential used to compute orbits is briefly described. In Section 3 we present our results with periodic orbit analysis, density response and phase space studies. Finally, in Section 4 we present our conclusions.

2. Methodology and Numerical Implementation

A common method to study the effect of spiral arms on stellar dynamics recurs to the use of elegant but simple 2-D bisymmetric local approximations such as cosine functions (TWA based), in this scheme, spiral arms are assumed to be smooth self-consistent perturbations to the axisymmetric potential. Cosine potentials to represent spiral arms, are in several cases taken beyond its self-consistent validity regime by imposing large pitch angles and/or large amplitudes for the spiral arms. However, in this regime, other methods to test undirectly self-consistency of steady potentials, like the construction of periodic orbits, are applicable. Details of a non-axisymmetric potential are not negligible when we are talking about a global model, about sensitive material as it is the gas, or about sensitive orbital behavior as chaos. Fine details of a complicate three dimensional distribution as spiral arms in galaxies are far beyond of an approximation as simple as a cosine function. Thus, we use a better approximation based on a 3-D model density distribution, that allows more complicate shapes and a more detailed representation of a potential for spiral arms.

We employ the spiral arms potential formed by oblate spheroids called `PERLAS` (**Sp**iral arms pot**e**ntial fo**r**med by ob**la**te **s**pheroids) from Pichardo et al. (2003), to represent their 3-D density based nature. This 3-D steady two-armed self-gravitating potential results to be more realistic in the sense that it considers the force exerted by the whole spiral arms structure, sculpting much more complicated shapes for the gravitational potential and gravitational force than a simple 2-D cosine function. This intrinsic difference gives rise to significant deviations on orbital dynamics when compared to a cosine potential. Comparisons of the model with observations and with other models have been already published (Pichardo et al. 2003; Martos et al. 2004; Antoja et al. 2009; Antoja et al. 2011).

The corresponding parameters used to produce models for late type spiral galaxies are presented in Table 1. Spiral arms self-consistency has been tested through the reinforcement of the spiral potential by stellar orbits (Patsis et al. 1991; Pichardo et al. 2003). The total mass of the spiral arms in our model is of 3% of the disc mass, which represents conservative (low mass) spiral arms for late type disc galaxies. We have employed the known parameter $Q_T(R)$ (Combes & Sanders 1981) applied to bars and spiral arms (Buta & Block 2001; Laurikainen & Salo 2002; Vorobyov 2006; Kalapotharakos et al. 2010, etc.), to

measure the strength of the spiral arms (`PERLAS`) in order to compare with observations and other models. We present in Figure 1 the maximum value, for models with different pitch angles, of the parameter $\mathrm{Q_T(R)}=\mathrm{F_T^{max}(R)}/|\langle \mathrm{F_R(R)}\rangle|$, where $\mathrm{F_T^{max}} = |\left(\frac{1}{R}\,\partial\Phi(\mathrm{R},\theta)/\partial\theta\right)|_{\mathrm{max}}$, represents the maximum amplitude of the tangential force at radius R, and $\langle \mathrm{F_R(R)}\rangle$, is the mean axisymmetric radial force at the same radius, derived from the m=0 component of the gravitational potential. Each point in this curve represents a different pitch angle, going from 0° to almost 90°. Maximum values up to 0.4 for this parameter are reasonable for late spirals (Buta et al. 2005).

The axisymmetric potential includes a Miyamoto-Nagai disk and bulge, and a massive halo (based on the potential described by Allen & Santillán 1991 for the Milky Way) to fit known observational parameters on late spirals: mainly rotation curves, mass ratios (between the components: bulge, disk and halo) and scale lengths. In Table 1, we present the employed axisymmetric and non-axisymmetric (spiral arms) parameters.

3. Results

In the case of long lasting steady potentials, self-consistency can be tested undirectly through the construction of periodic orbits. The existence of periodic orbits supporting a large scale structure such as spiral arms increases the probability of mantaining long lasting large scale structures. We present a periodic orbit study in Figure 2. In order to quantify the support of periodic orbits to the spiral arm potential, we follow the method of Contopoulos & Grosbøl (1986) to obtain the density response to the given spiral perturbation. This method assumes that the stars with orbits trapped around an unperturbed circular orbit, and with the sense of rotation of the spiral perturbation, are also trapped around the corresponding central periodic orbit in the presence of the perturbation. In this manner, we computed a series of central periodic orbits and found the density response along their extension, using the conservation of mass flux between any two successive orbits. We found the position of the density response maxima (filled squares in Figure 2) along each periodic orbit, thus the positions of the response maxima on the galactic plane are known. These positions are compared with the center of the imposed spiral arms potential, i.e. the spiral locus (open squares in Figure 2).

For smaller pitch angles ($i \lesssim 20°$), the density response maxima support closely the spiral arms potential, making the existence of long lasting spiral arms more probable. On the other hand, for spiral arms with pitch angles larger than $\sim 20°$, the response maxima precede systematically the spiral arms potential, i.e. the response produces spiral arms with much smaller pitch angles than the spiral arms potential. The response (or support) “avoids”

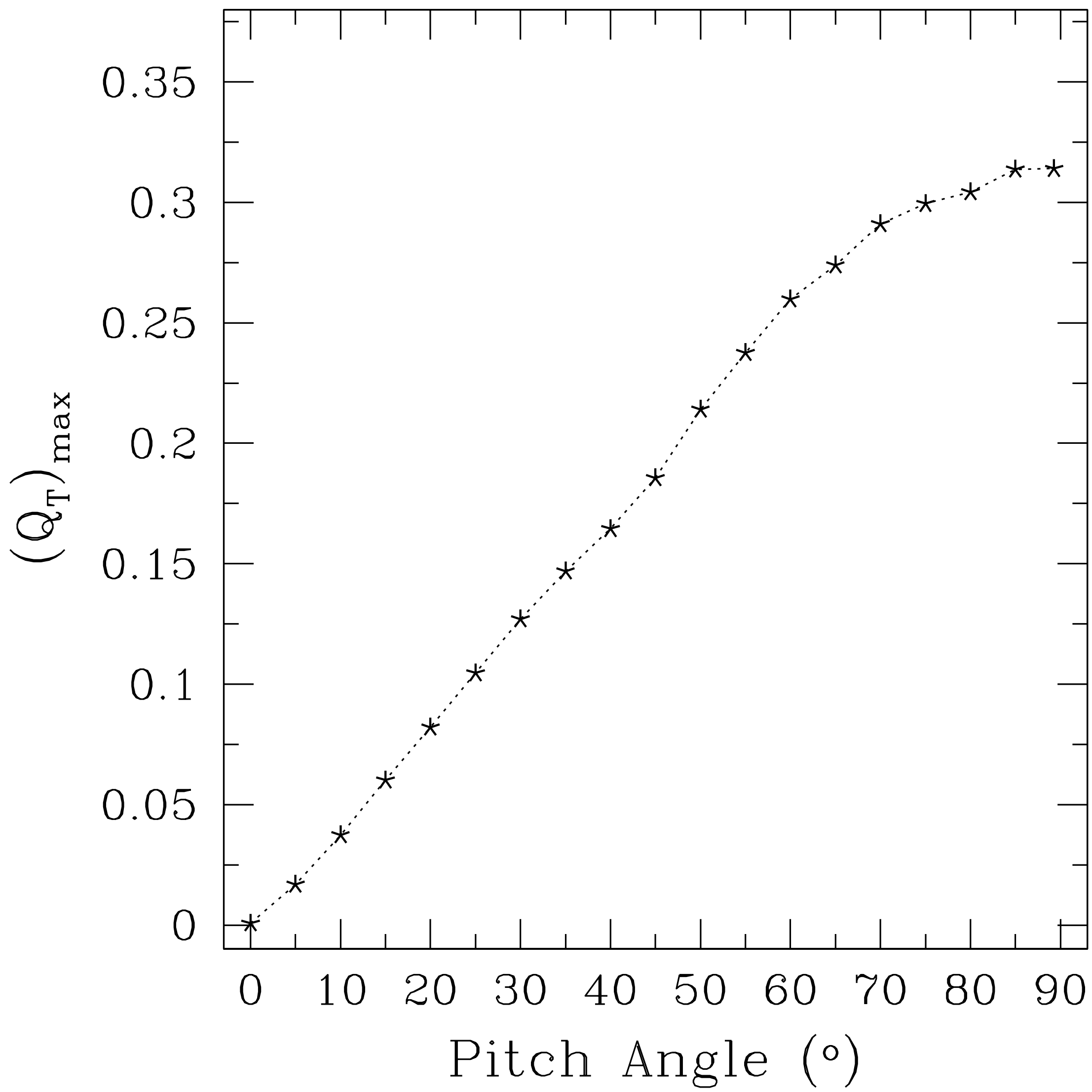

Fig. 1.— $(\mathrm{Q_T})_{\mathrm{max}}$ represents the maximum value of the parameter $\mathrm{Q}_T(R)$ (maximum relative torques) vs. pitch angle of the spiral arms model.

Table 1. Parameters of the Spiral Arms Models

Parameter	Value	Reference
	Spiral Arms	
locus	Logarithmic	1,10
arms number	2	2
pitch angle	10-60°	3,8
M_{Spiral}/M_{disk}	3%	
scale-length	disk based: 3 kpc	4,5
Radial force contrast	5-10%	6
pattern speed (Ω_{sp})	-20 (clockwise) $\mathrm{km\,s^{-1}kpc^{-1}}$	1,7
ILR position	2.03 kpc	
Corotation position	8.63 kpc	
inner limit	2.03 kpc	∼ILR position based
external limit	8.63 kpc	∼corotation position based
	Axisymmetric Components	
Disk Mass / Halo Mass	0.1 (up to 100 kpc halo radius)	4,9
Bulge Mass / Disc Mass	0.2	5,9
Rot. Curve (V_{max})	170 $\mathrm{km\,s^{-1}}$	8
Disk Mass	$5.10\times^{10}$ $M_\odot$	4
Bulge Mass	$1.02\times^{10}$ $M_\odot$	M_D/M_B based
Halo Mass	$4.85\times^{11}$ $M_\odot$	M_D/M_H based
Disk scale-length	3 kpc	4,5

References. — 1) Grosbol & Patsis 1998. 2) Drimmel et al. 2000. 3) Kennicutt 1981. 4) Pizagno et al. 2005 5) Weinzirl et al. 2009. 6) Contopoulos 2007. 7) Patsis et al. 1991; Fathi et al. 2009. 8) Ma et al. 2000; Brosche 1971; Sofue & Rubin 2001. 9) Block et al. 2002. 10) Pichardo et al. 2003.

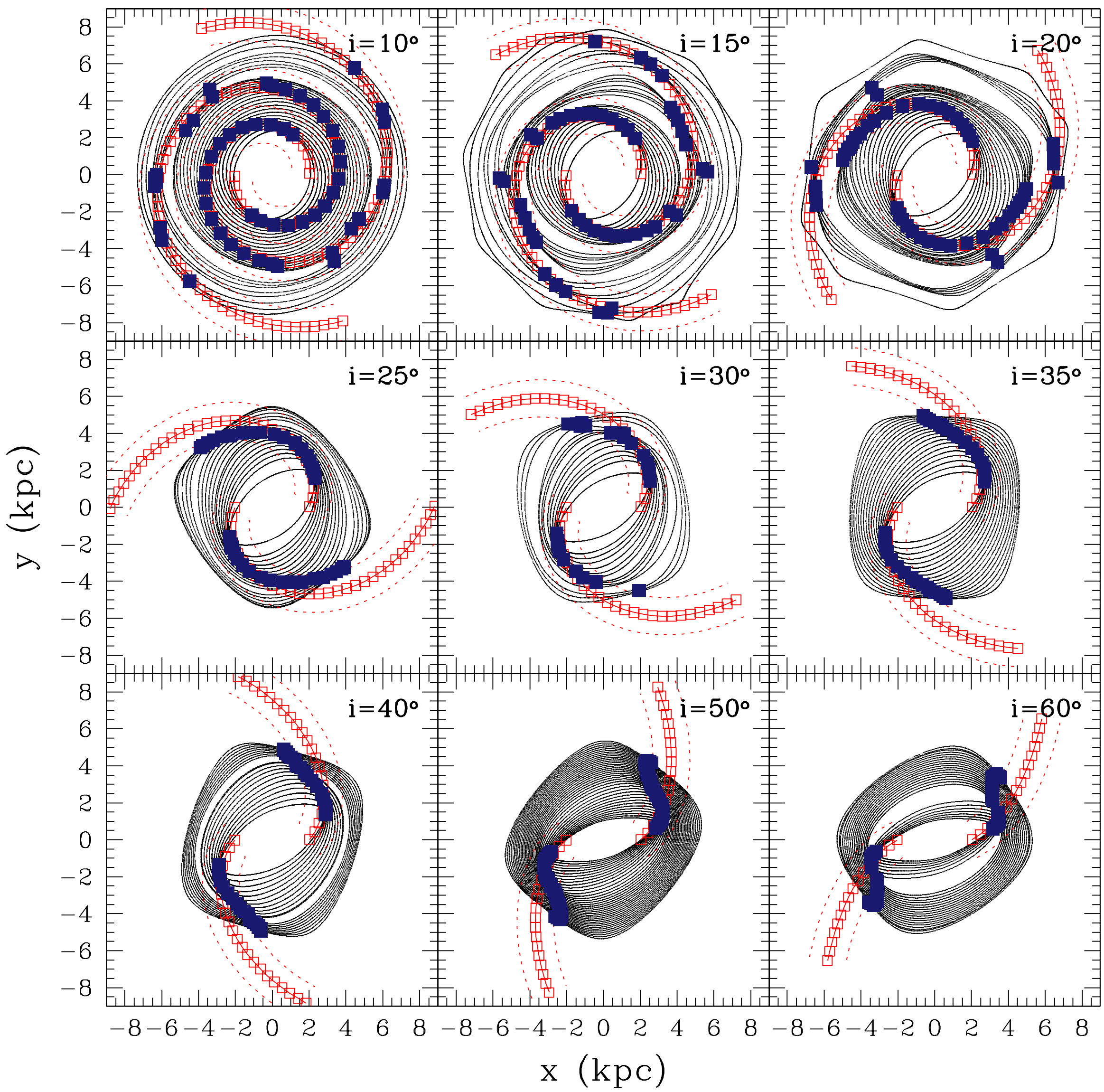

Fig. 2.— Periodic orbits and response maxima (filled squares) in models with the spiral loci (open squares) for the 3D spiral model (Table 1), with pitch angles ranging from 10 to 60 °.

open long lasting spiral arms. Beyond 20° of pitch angle for the spiral potential, the density response keeps almost the same pitch angle (approximately 18° to 22°) independently of the potential. Long lasting spirals are not supported anymore, spiral arms in this case may be rather transient.

In Figure 3 we present a 3×3 phase space diagrams mosaic to show the main results. These 9 panels show Poincaré diagrams with different Jacobi energy families running from Ej=-1080 to -1010 $\times 10^2$ km^2 s^{-2}, covering the total extension of the spiral arms, as we go from 30° (upper line of diagrams) to 50° (bottom line of diagrams). The left part of each diagram represents prograde orbits in the reference system of the spiral arms.

Observations of spiral galaxies show pitch angles up to $\sim 50°$ (Ma et al. 2000). Despite that periodic orbits are not supporting spiral arms, the existence of very open spirals, could indicate a probable transient nature. However, even then, some ordered orbits are expected to support for short periods these large scale structures.

We have studied the effect of increasing the pitch angle in a typical late type spiral galaxy model. With this study we search for a limit for the pitch angle, for which chaos becomes pervasive and destroys all ordered orbits in the relevant spiral arms region. At a pitch angle of 20° or less, the majority of orbits are ordered and simple, periodic orbits support spiral arms up to close to corotation. As we increase the pitch angle, at 30° (first line of diagrams from the top of Figure 3), the orbital behavior is much more complex, presenting resonant islands and the onset of chaos is clear, surrounding the stable periodic orbits, yet supporting them in a contained region. For 40° (second line of diagrams), chaos becomes pervasive compromising the available phase space around the stable periodic orbits. For 50° (bottom line of diagrams), the chaotic region covers almost all regular prograde orbits, approaching closely to the main periodic orbits. For pitch angles beyond $\sim 50°$ chaos destroys periodic orbits.

In all cases reported in this work, the radial position of corotation was kept fixed (see Table 1). However, taking into account earlier work by Contopoulos & Grosbøl (1986); Patsis, Contopoulos & Grosbøl (1991), among others, we produced several experiments taking the position of corotation (to 16.5 kpc by reducing the spiral arms angular speed from 20 to 10 km s^{-1} kpc^{-1}) considerably far from the end of the spiral arms to check whether it is relevant in the observed chaotic behavior. Although a large fraction of chaos is indeed produced close to the corotation resonance, we found that even with corotation far from the end of spiral arms, a fraction of chaos is generated toward larger pitch angles, although not enough to compromise ordered regions around the periodic orbits. We then produced experiments where we fixed the end of the spiral arms close to the corotation position, and changed only one pararameter: the pitch angle. We found then that the chaos produced in corotation

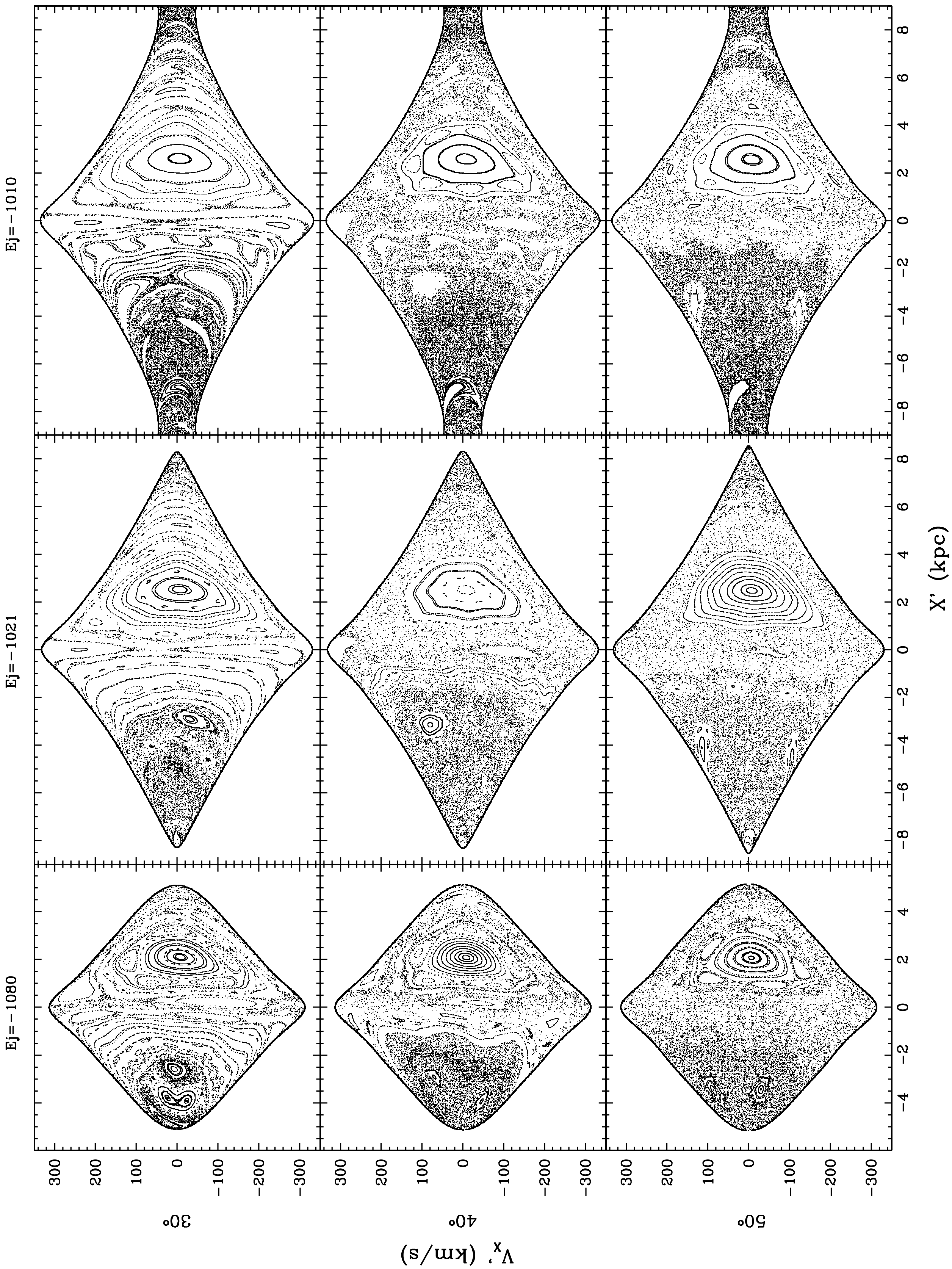

Fig. 3.— Phase space diagrams with $E_J = [-1080, -1010]$, in units of 10^2 km^2 s^{-2}. From upper to bottom lines of diagrams, pitch angles go from 30° to 50°.

becomes much stronger as we increase the pitch angle. Chaos seems a combination between the effect of the corotation resonance and the pitch angle.

4. Conclusions

With the use of an axisymmetric fixed model to simulate a typical late type galaxy as a background, we superposed a bisymmetric steady spiral arm potential (`PERLAS`) and studied the evolution of orbital behavior in the plane of the disk, as we change the pitch angle going from 10° to 60°, in order to set some structural restrictions to the spiral arms, based on orbital dynamics. Observed galaxies classified as late type spirals present a wide scatter in pitch angles, going from $\sim 10^\circ$ to 50°. With these family of models, we have carried out an exhaustive orbital study (order and chaos) with periodic orbits and with phase space diagrams.

In this paper we present the first restriction relative to the pitch angle. In the case of ordered motion, with periodic orbits, a limit in the pitch angle of the density response is found at approximately 20°, up to which, the density response reinforces the spiral arms potential at all radii, i.e. with a more long-lasting nature. Beyond this limit, the density response "avoids" to follow the spiral arm potential, producing pitch angles much smaller than the background spiral potential. Spiral arms beyond this limit might be better explained as transient structures.

A second restriction is obtained out of chaos behavior. With the phase space orbital study, going from pitch angles of 10°, where order reigns, to more than 50°, where chaos becomes pervasive. With this orbital study we are able to set a limit value for the maximum pitch angle before the system becomes completely chaotic. This limit closely coincides with the observational maximum pitch angle of spirals ($\sim 50^\circ$), suggesting a possible relation between the structural characteristics of the galaxy and chaos.

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